AQA

Please write clearly in block capitals.

Centre number	
Candidate number	
Surname	ANSWERS
Forename(s)	
Candidate signature	

AS MATHEMATICS

Unit Decision 1

A

Friday 24 June 2016 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

JUN16MD0101

PB/Jun16/E4

MD01

Answer **all** questions.

Answer each question in the space provided for that question.

1 Alfred has bought six different chocolate bars. He wants to give a chocolate bar to each of his six friends. The table gives the names of the friends and indicates which of Alfred's six chocolate bars they like.

Friend	Chocolate Bar
Flavio	Coffee, Nut
Ghania	Lemon, Mint
Harry	Lemon, Mint, Orange
Imogen	Mint
Jenny	Nut, Plain
Kim	Orange, Plain

(a) Draw a bipartite graph to represent this information.

[2 marks]

(b) Alfred makes an initial matching of his friends and the chocolate bar they will each receive:

Ghania – Mint Jenny – Plain Harry – Lemon

Kim – Orange Flavio – Nut

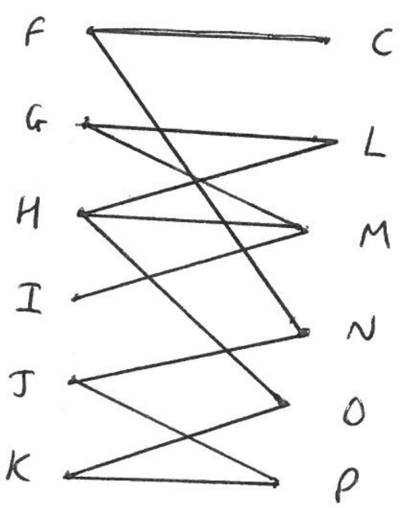
Demonstrate, by using an alternating-path algorithm from this initial matching, how each of the friends can be given a chocolate bar that they like.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 1

(a)



QUESTION PART REFERENCE

Answer space for question 1

b) GM, JP, HL, KO, FN

I − M + G − L + H − O + K − P + J − N + F − C

I + M − G + L − H + O − K + P − J + N − F + C

FC, GL, HO, IM, JN, KP

Turn over ▶

0 3

2 (a) Use a shuttle sort to rearrange into alphabetical order the following list of names:

Rob, Eve, Meg, Ian, Xavi

Show the list at the end of each pass.

[3 marks]

(b) A list of **ten** numbers is sorted into ascending order, using a shuttle sort.

(i) How many passes are needed?

(ii) Give the maximum number of comparisons needed in the sixth pass.

(iii) Given that the list is initially in descending order, find the total number of swaps needed.

[4 marks]

QUESTION PART REFERENCE	Answer space for question 2

a)

	1st		2nd			3rd	4th
R	E	E	E	E	E	E	E
E	R	M	M	M	I	I	I
M	M	R	R	I	M	M	M
I	I	I	I	R	R	R	R
X	X	X	X	X	X	X	X

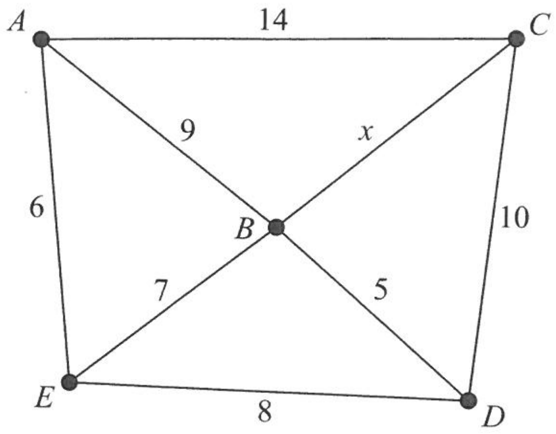
bi) $\underline{9}$ passes needed for 10 numbers

ii) $\underline{6}$ comparisons (starts with 6th + 7th numbers moving up to 1st and 2nd)

ii) $1+2+3+4+5+6+7+8+9 = \underline{45}$

(maximum no. of comparisons per pass up to 9 passes) tside box

3 The network below shows vertices A, B, C, D and E. The number on each edge shows the distance between vertices.



(a) (i) In the case where $x = 8$, use Kruskal's algorithm to find a minimum spanning tree for the network. Write down the order in which you add edges to your minimum spanning tree.

(ii) Draw your minimum spanning tree.

(iii) Write down the length of your minimum spanning tree.

[4 marks]

(b) Alice draws the same network but changes the value of x. She correctly uses Kruskal's algorithm and edge CD is included in her minimum spanning tree.

(i) Explain why x cannot be equal to 7.

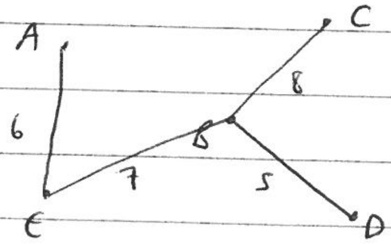
(ii) Write down an inequality for x.

[3 marks]

QUESTION PART REFERENCE | **Answer space for question 3**

3ai) BD(5)
AE(6)
BE(7)
BC(8)

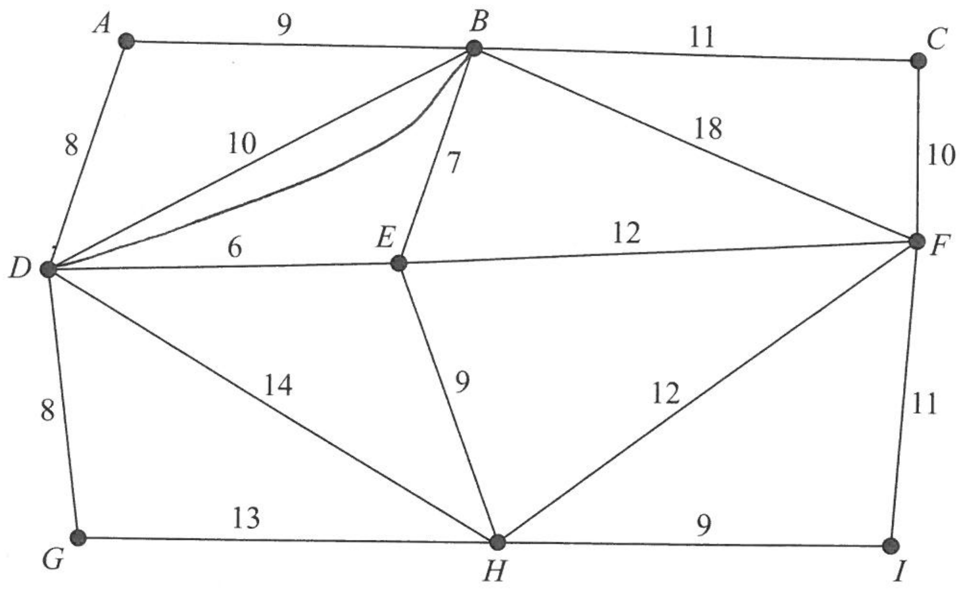
ii)



iii) $5+6+7+8 = \underline{26}$

QUESTION PART REFERENCE	Answer space for question 3
b i)	if $x = 7$, BC would have to be chosen before CD and then CD would create a cycle
ii)	$x \geq 10$ (BC or CD could then be included)

4 Amal delivers free advertiser magazines to all the houses in his village. The network shows the roads in his village. The number on each road shows the time, in minutes, that Amal takes to walk along that road.



Total of all times $= 167$ minutes

(a) Amal starts his delivery round from his house, at vertex A. He must walk along each road at least once.

(i) Find the length of an optimal Chinese postman route around the village, starting and finishing at Amal's house.

(ii) State the number of times that Amal passes his friend Dipak's house, at vertex D.

[6 marks]

(b) Dipak offers to deliver the magazines while Amal is away on holiday. Dipak must walk along each road at least once. Assume that Dipak takes the same length of time as Amal to walk along each road.

(i) Dipak can start his journey at **any** vertex and finish his journey at **any** vertex. Find the length of time for an optimal route for Dipak.

(ii) State the vertices at which Dipak could finish, in order to achieve his optimal route.

[3 marks]

(c) (i) Find the length of time for an optimal route for Dipak, if, instead, he wants to finish at his house, at vertex D, and can start his journey at any other vertex.

(ii) State the start vertex.

[2 marks]

QUESTION PART REFERENCE	Answer space for question 4
4ai)	odd vertices : B, D, F, H
	BD (10) BD + FH = 10 + 12 = 22
	BF (18) BF + DH = 18 + 14 = 32
	BH (16) (BEH) BH + DF = 16 + 18 = 34
	DF (18) (DEF)
	DH (14)
	FH (12) 167 + 22 = 189
aii)	$D \to \frac{6}{2} = 3$ (as BD is repeated)
bi)	Only repeat BD as smallest edge at 10
	∴ start at F and finish at H
	167 + 10 = 177
ii)	F and H
ci)	Must now repeat FH in order to finish at D.
	167 + 12 = 179
ii)	B

5 A fair comes to town one year and sets up its rides in two large fields that are separated by a river. The diagram shows the ten main rides, at A, B, C, …, J. The numbers on the edges represent the times, in minutes, it takes to walk between pairs of rides. A footbridge connects the rides at D and F.

(a) (i) Use Dijkstra's algorithm on the diagram below to find the minimum time to walk from A to each of the other main rides.

(ii) Write down the route corresponding to the minimum time to walk from A to G.

[5 marks]

(b) The following year, the fair returns. In addition to the information shown on the diagram, another footbridge has been built to connect the rides at E and G. This reduces the time taken to travel from A to G, but the time taken to travel from A to J is not reduced.

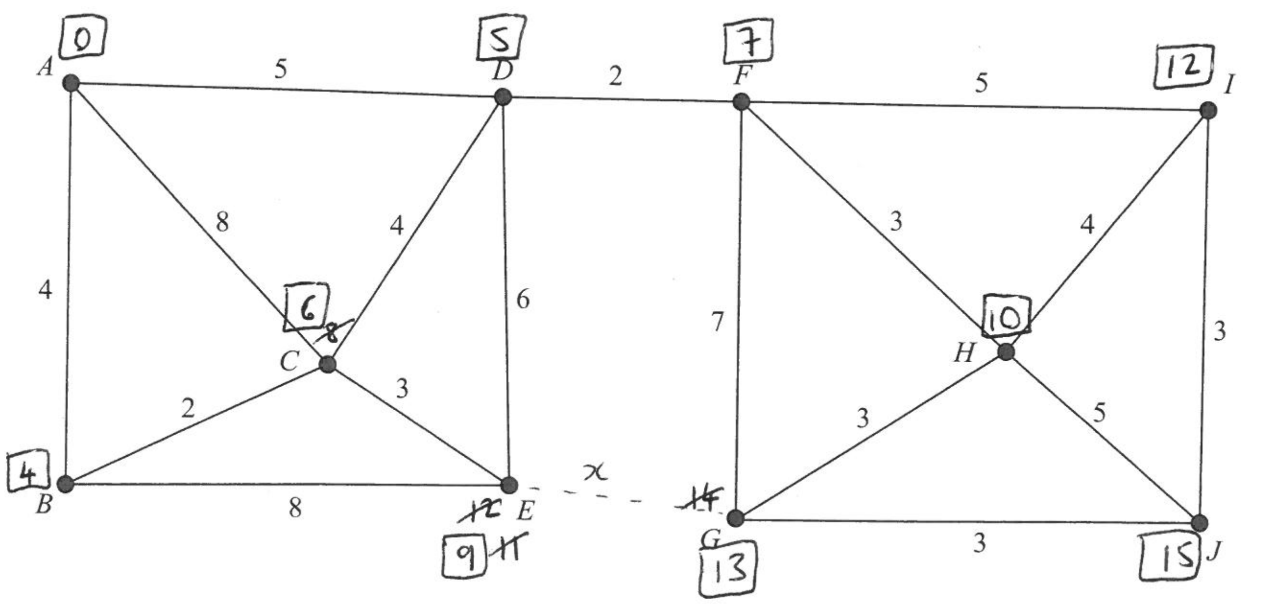
The time to walk across the footbridge from E to G is x minutes, where x is an integer.

Find two inequalities for x and hence state the value of x.

[3 marks]

Answer space for question 5

(a)(i)



QUESTION PART REFERENCE	Answer space for question 5
ii)	A D F H G
b)	$A \to G < 13$ (time reduced)
	$A \to J \geq 15$ (time <u>not</u> reduced)
	$A \to E \to G = 9 + x$
	$\therefore 9 + x < 13$
	$\underline{x < 4}$
	$A \to E \to G \to J = 9 + x + 3$
	$\therefore 9 + x + 3 \geq 15$
	$x + 12 \geq 15$
	$x \geq 3$
	so, $x < 4$ and $x \geq 3$
	so, $\underline{\underline{x = 3}}$

Turn over ▶

6 A connected graph is semi-Eulerian if exactly two of its vertices are of odd degree.

(a) A graph is drawn with 4 vertices and 7 edges. What is the sum of the degrees of the vertices?

[1 mark]

(b) Draw a simple semi-Eulerian graph with exactly 5 vertices and 5 edges, in which exactly one of the vertices has degree 4.

[2 marks]

(c) Draw a simple semi-Eulerian graph with exactly 5 vertices that is also a tree.

[2 marks]

(d) A simple graph has 6 vertices. The graph has two vertices of degree 5.

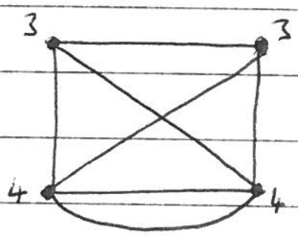
Explain why the graph can have no vertex of degree 1.

[2 marks]

QUESTION PART REFERENCE

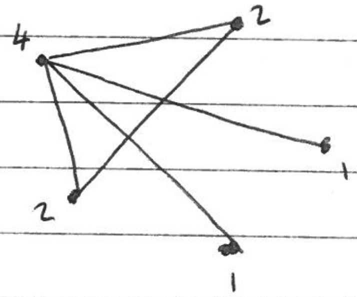
Answer space for question 6

6a)

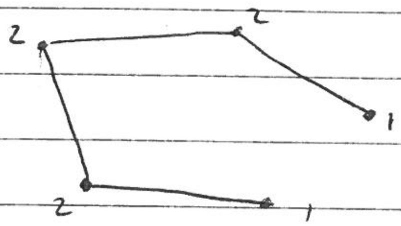


$3+3+4+4 = \underline{14}$

b)



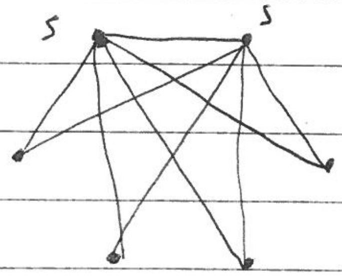
c)



QUESTION PART REFERENCE

Answer space for question 6

d)



6 vertices with two having degree of 5. since it is simple (no loops), then 2 vertices must be connected to all other vertices. ∴ not vertex can have just one edge ie, degree 1. The minimum no of edges from each vertex will be 2 ie, degree 2

7 A company operates a steam railway between six stations. The minimum cost (in euros) of travelling between pairs of stations is shown in the table below.

(a) **On Figure 1** below, use Prim's algorithm, starting from P, to find a minimum spanning tree for the graph connecting P, Q, R, S, T and U. State clearly the order in which you select the vertices and draw your minimum spanning tree.

[6 marks]

Question 7 continues on page 20

QUESTION PART REFERENCE

Answer space for question 7(a)

Figure 1

	P	Q	R	S	T	U
P	–	14	7	11	6	12
Q	14	–	8	10	9	10
R	7	8	–	12	13	15
S	11	10	12	–	5	11
T	6	9	13	5	–	10
U	12	10	15	11	10	–

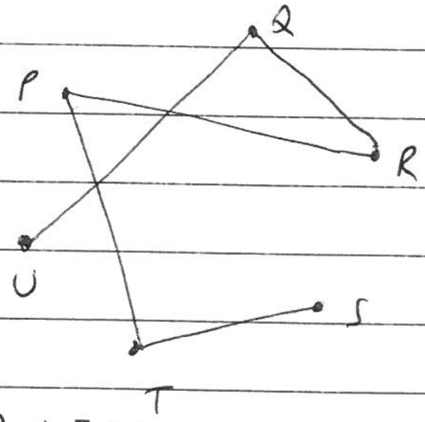
7a) PT (6)

ST (5)

PR (7)

RQ (8)

QU (10)



Total length = 6 + 5 + 7 + 8 + 10

= 36

(b) Another station, V, is opened. The minimum costs (in euros) of travelling to and from V to each of the other stations are added to the table in part **(a)**, as shown in **Figure 2(i)** below. Further copies of this table are shown in **Figure 2(ii)**.

Arjen is on holiday and he plans to visit each station. He intends to board a train at V and visit all the other stations, once only, before returning to V.

(i) By first removing V, obtain a lower bound for the minimum travelling cost of Arjen's tour. (You may use **Figure 2(i)** for your working.)

[3 marks]

(ii) Use the nearest neighbour algorithm **twice**, starting each time from V, to find two different upper bounds for the minimum cost of Arjen's tour. State, with a reason, which of your two answers gives the better upper bound. (You may use **Figure 2(ii)** for your working.)

[6 marks]

(iii) Hence find an optimal tour of the seven stations. Explain how you know that it is optimal.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 7(b)

Figure 2(i)

	P	Q	R	S	T	U	V
P	–	14	7	11	6	12	15
Q	14	–	8	10	9	10	18
R	7	8	–	12	13	15	14
S	11	10	12	–	5	11	14
T	6	9	13	5	–	10	17
U	12	10	15	11	10	–	12
V	15	18	14	14	17	12	–

b i) VU (12) + VR (14) = 26

TS (5) + TP (6) + PR (7) + RQ (8) + QU (10) = 36

lower bound = 26 + 36 = <u>62</u>

QUESTION PART REFERENCE

Answer space for question 7(b)

Figure 2(ii)

	P	*Q*	*R*	*S*	*T*	*U*	*V*
P	–	14	7	11	6	12	15
Q	14	–	8	10	9	10	18
R	7	8	–	12	13	15	14
S	11	10	12	–	5	11	14
T	6	9	13	5	–	10	17
U	12	10	15	11	10	–	12
V	15	18	14	14	17	12	–

	P	*Q*	*R*	*S*	*T*	*U*	*V*
P	–	14	7	11	6	12	15
Q	14	–	8	10	9	10	18
R	7	8	–	12	13	15	14
S	11	10	12	–	5	11	14
T	6	9	13	5	–	10	17
U	12	10	15	11	10	–	12
V	15	18	14	14	17	12	–

$V \to U \to Q \to R \to P \to T \to S \to V$

bii) $VU(12) + UQ(10) + QR(8) + RP(7) + PT(6) + TS(5) + SV(14)$

$12 + 10 + 8 + 7 + 6 + 5 + 14 = \underline{62}$

$V \to U \to T \to S \to Q \to R \to P \to V$

$12 + 10 + 5 + 10 + 8 + 7 + 15 = \underline{67}$

62 is best upper bound as both give a tour but 62 is lower than 67.

Turn over ▶

QUESTION PART REFERENCE	Answer space for question 7(b)
iii)	lower bound → $T \geqslant 62$
	upper bound → $T \leqslant 62$
	$62 \leqslant T \leqslant 62$
	∴ optimal tour is 62
	VUQRPTSV

8 Nerys runs a cake shop. In November and December she sells Christmas hampers. She makes up the hampers herself, in two sizes: Luxury and Special.

Each day, Nerys prepares x Luxury hampers and y Special hampers.

It takes Nerys 10 minutes to prepare a Luxury hamper and 15 minutes to prepare a Special hamper. She has 6 hours available, each day, for preparing hampers.

10x 15y 360mins

From past experience, Nerys knows that each day:

- she will need to prepare at least 5 hampers of each size
- she will prepare at most a total of 32 hampers
- she will prepare at least twice as many Luxury hampers as Special hampers.

Each Luxury hamper that Nerys prepares makes her a profit of £15; each Special hamper makes a profit of £20. Nerys wishes to maximise her daily profit, £P.

(a) Show that x and y must satisfy the inequality $2x + 3y \leqslant 72$.

[1 mark]

(b) In addition to $x \geqslant 5$ and $y \geqslant 5$, write down two more inequalities that model the constraints above.

[2 marks]

(c) On the grid opposite draw a suitable diagram to enable this problem to be solved graphically. Indicate a feasible region and the direction of an objective line.

[7 marks]

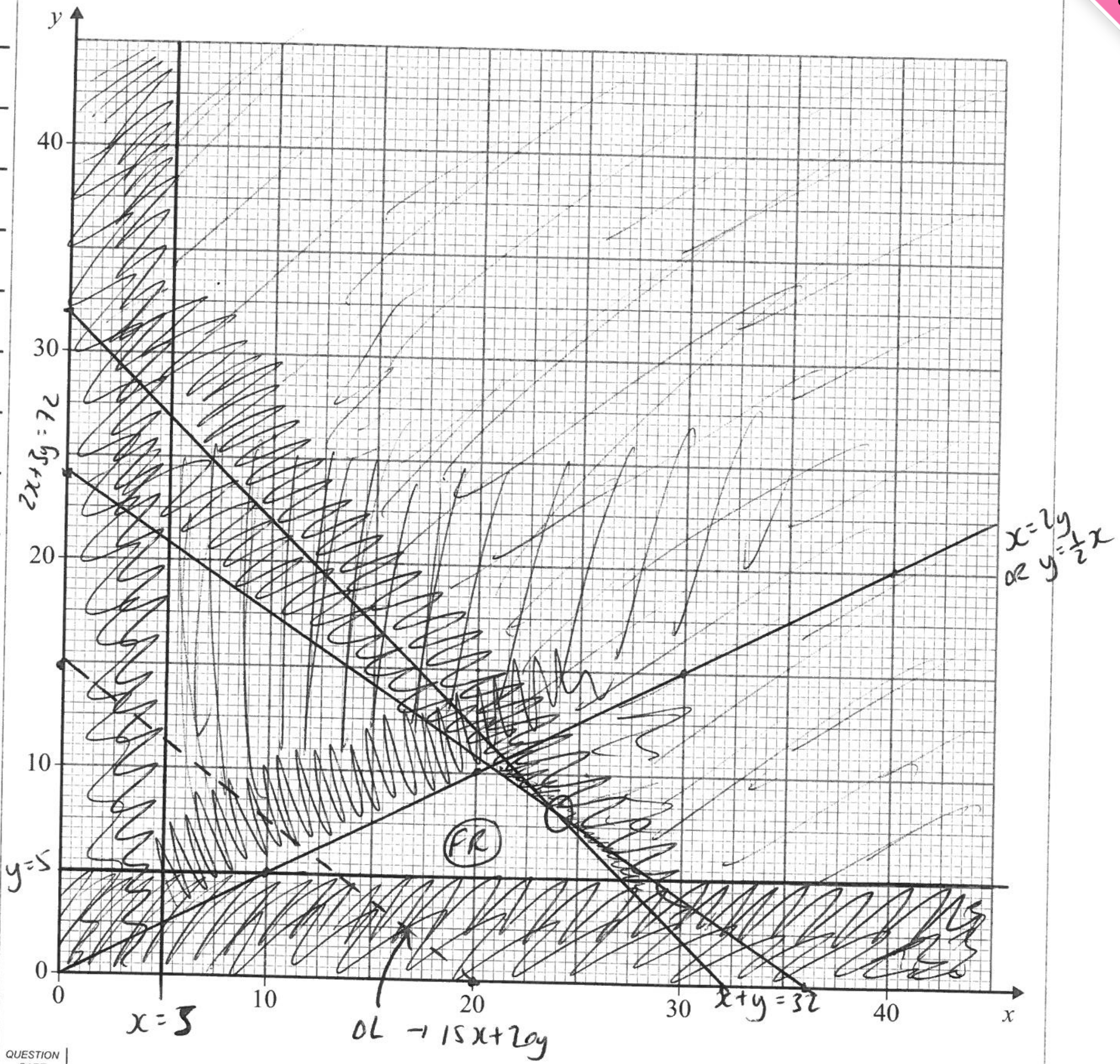
(d) (i) Use your diagram to find the number of each type of hamper that Nerys should prepare each day to achieve a maximum profit.

(ii) Calculate this profit.

[3 marks]

QUESTION PART REFERENCE	Answer space for question 8
8a)	6hours = 360mins available at most
	luxury hamper (x) → 10mins per x → 10x
	special hamper (y) → 15mins per y → 15y
	so, $10x + 15y \leq 360$ $(\div 5)$
	$2x + 3y \leq 72$ (as req)
b)	$x + y \leq 32$ → 32 hampers at most
	$x \geqslant 2y$ → at least twice as many ~~special~~ luxury as special

Answer space for question 8



QUESTION PART REFERENCE

c) $P = 15x + 20y$ (£15 profit for luxury, £20 profit for standard)

maximise

(plot 15 on y and 20 on x)

di) maximise at intersection of $x+y=32$ and $2x+3y=72$

$2x+2y=64$

$2x+16=64$

$x=24$ (luxury)

$y=8$ (special)

Turn over ▶

QUESTION PART REFERENCE	Answer space for question 8
ii)	$P = 15x + 20y$, $x = 24$, $y = 8$
	$P = 15(24) + 20(8)$
	$= 360 + 160$
	$= £520$ profit

END OF QUESTIONS

There are no questions printed on this page

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